# EXHIBIT Y

## Exhibit A (Supplemental)

U.S. Pat. No. 9,218,156 Claim 7

#### SUPPLEMENTAL INFRINGEMENT EVIDENCE

## 7. A device comprising:

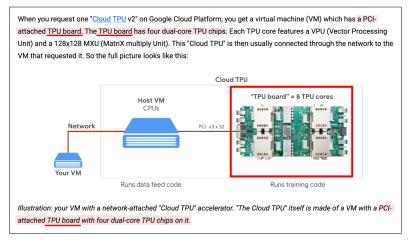
at least one first low precision high-dynamic range (LPHDR) execution unit adapted to execute a first operation on a first input signal representing a first numerical value to produce a first output signal representing a second numerical value,

wherein the dynamic range of the possible valid inputs to the first operation is at least as wide as from 1/1,000,000 through 1,000,000 and for at least X=5% of the possible valid inputs to the first operation, the statistical mean, over repeated execution of the first operation on each specific input from the at least X% of the possible valid inputs to the first operation, of the numerical values represented by the first output signal of the LPHDR unit executing the first operation on that input differs by at least Y=0.05% from the result of an exact mathematical calculation of the first operation on the numerical values of that same input;

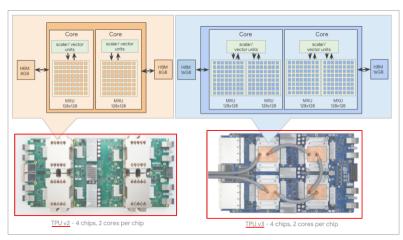
at least one first computing device adapted to control the operation of the at least one first LPHDR execution unit

wherein the at least one first computing device comprises at least one of a central processing unit (CPU), a graphics processing unit (GPU), a field programmable gate array (FPGA), a microcode-based processor, a hardware sequencer, and a state machine; and,

wherein the number of LPHDR execution units in the device exceeds by at least one hundred the non-negative integer number of execution units in the device adapted to execute at least the operation of multiplication on floating point numbers that are at least 32 bits wide. As demonstrated below, the Accused Products include multiple components that, separately and independently, meet all the requirements of the claimed "device.":



https://codelabs.developers.google.com/codelabs/keras-flowers-convnets/#21



https://cloud.google.com/tpu/docs/system-architecture

<sup>&</sup>lt;sup>1</sup> Unless indicated otherwise, color-coded annotations have been added in order to identify relevant components and features of the Accused Products.

#### SUPPLEMENTAL INFRINGEMENT EVIDENCE

## 7. A **device** comprising:

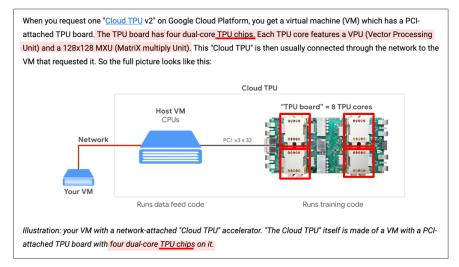
at least one first low precision high-dynamic range (LPHDR) execution unit adapted to execute a first operation on a first input signal representing a first numerical value to produce a first output signal representing a second numerical value,

wherein the dynamic range of the possible valid inputs to the first operation is at least as wide as from 1/1,000,000 through 1,000,000 and for at least X=5% of the possible valid inputs to the first operation, the statistical mean, over repeated execution of the first operation on each specific input from the at least X% of the possible valid inputs to the first operation, of the numerical values represented by the first output signal of the LPHDR unit executing the first operation on that input differs by at least Y=0.05% from the result of an exact mathematical calculation of the first operation on the numerical values of that same input;

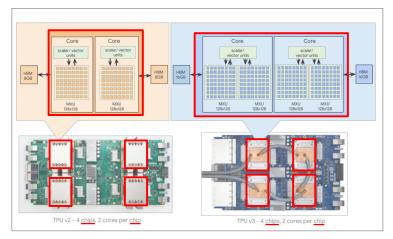
at least one first computing device adapted to control the operation of the at least one first LPHDR execution unit

wherein the at least one first computing device comprises at least one of a central processing unit (CPU), a graphics processing unit (GPU), a field programmable gate array (FPGA), a microcode-based processor, a hardware sequencer, and a state machine; and,

wherein the number of LPHDR execution units in the device exceeds by at least one hundred the non-negative integer number of execution units in the device adapted to execute at least the operation of multiplication on floating point numbers that are at least 32 bits wide. As demonstrated below, the Accused Products include multiple components that, separately and independently, meet all the requirements of the claimed "device." For example, a "TPU Chip" satisfies these requirements:



https://codelabs.developers.google.com/codelabs/keras-flowers-convnets/#22



https://cloud.google.com/tpu/docs/system-architecture

See also generally Norrie et al., "Google's Training Chips Revealed: TPUv2 and TPUv3" (Presented at HotChips Conference, Aug. 2020)

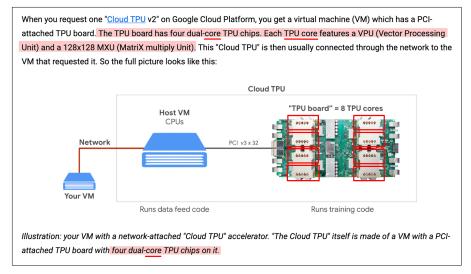
<sup>&</sup>lt;sup>2</sup> Unless indicated otherwise, color-coded annotations have been added to the figures in this chart to highlight relevant teachings of the prior art.

#### SUPPLEMENTAL INFRINGEMENT EVIDENCE

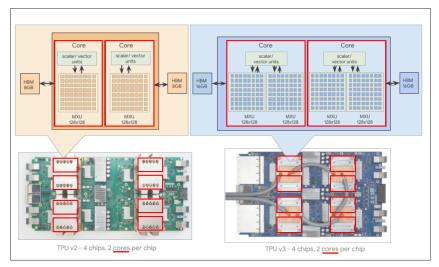
## 7. A **device** comprising:

- at least one first low precision high-dynamic range (LPHDR) execution unit adapted to execute a first operation on a first input signal representing a first numerical value to produce a first output signal representing a second numerical value,
- wherein the dynamic range of the possible valid inputs to the first operation is at least as wide as from 1/1,000,000 through 1,000,000 and for at least X=5% of the possible valid inputs to the first operation, the statistical mean, over repeated execution of the first operation on each specific input from the at least X% of the possible valid inputs to the first operation, of the numerical values represented by the first output signal of the LPHDR unit executing the first operation on that input differs by at least Y=0.05% from the result of an exact mathematical calculation of the first operation on the numerical values of that same input;
- at least one first computing device adapted to control the operation of the at least one first LPHDR execution unit
- wherein the at least one first computing device comprises at least one of a central processing unit (CPU), a graphics processing unit (GPU), a field programmable gate array (FPGA), a microcode-based processor, a hardware sequencer, and a state machine; and,
- wherein the number of LPHDR execution units in the device exceeds by at least one hundred the non-negative integer number of execution units in the device adapted to execute at least the operation of multiplication on floating point numbers that are at least 32 bits wide.

As demonstrated below, the Accused Products include multiple components that, separately and independently, meet all the requirements of the claimed "device." For example, a "TPU Core" satisfies these requirements:



https://codelabs.developers.google.com/codelabs/keras-flowers-convnets/#2

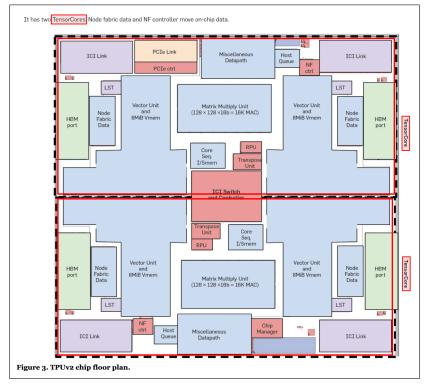


https://cloud.google.com/tpu/docs/system-architecture

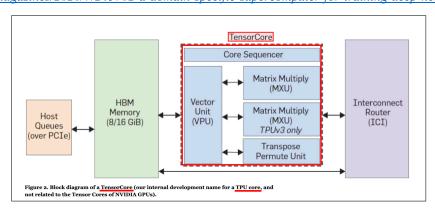
## 7. A **device** comprising:

- at least one first low precision high-dynamic range (LPHDR) execution unit adapted to execute a first operation on a first input signal representing a first numerical value to produce a first output signal representing a second numerical value,
- wherein the dynamic range of the possible valid inputs to the first operation is at least as wide as from 1/1,000,000 through 1,000,000 and for at least X=5% of the possible valid inputs to the first operation, the statistical mean, over repeated execution of the first operation on each specific input from the at least X% of the possible valid inputs to the first operation, of the numerical values represented by the first output signal of the LPHDR unit executing the first operation on that input differs by at least Y=0.05% from the result of an exact mathematical calculation of the first operation on the numerical values of that same input;
- at least one first computing device adapted to control the operation of the at least one first LPHDR execution unit
- wherein the at least one first computing device comprises at least one of a central processing unit (CPU), a graphics processing unit (GPU), a field programmable gate array (FPGA), a microcode-based processor, a hardware sequencer, and a state machine; and,
- wherein the number of LPHDR execution units in the device exceeds by at least one hundred the non-negative integer number of execution units in the device adapted to execute at least the operation of multiplication on floating point numbers that are at least 32 bits wide.

#### SUPPLEMENTAL INFRINGEMENT EVIDENCE



https://cacm.acm.org/magazines/2020/7/245702-a-domain-specific-supercomputer-for-training-deep-neural-networks



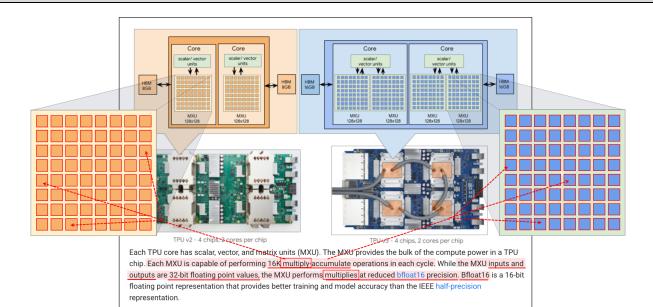
Id.

See also GOOG-SING-SC-000001-454.

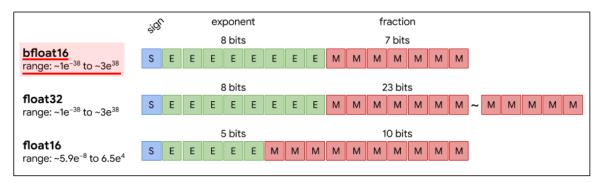
## 7. A device comprising:

- at least one first low precision high-dynamic range (LPHDR) execution unit adapted to execute a first operation on a first input signal representing a first numerical value to produce a first output signal representing a second numerical value,
- wherein the dynamic range of the possible valid inputs to the first operation is at least as wide as from 1/1,000,000 through 1,000,000 and for at least X=5% of the possible valid inputs to the first operation, the statistical mean, over repeated execution of the first operation on each specific input from the at least X% of the possible valid inputs to the first operation, of the numerical values represented by the first output signal of the LPHDR unit executing the first operation on that input differs by at least Y=0.05% from the result of an exact mathematical calculation of the first operation on the numerical values of that same input;
- at least one first computing device adapted to control the operation of the at least one first LPHDR execution unit
- wherein the at least one first computing device comprises at least one of a central processing unit (CPU), a graphics processing unit (GPU), a field programmable gate array (FPGA), a microcode-based processor, a hardware sequencer, and a state machine; and,
- wherein the number of LPHDR execution units in the device exceeds by at least one hundred the non-negative integer number of execution units in the device adapted to execute at least the operation of multiplication on floating point numbers that are at least 32 bits wide.

#### SUPPLEMENTAL INFRINGEMENT EVIDENCE



#### https://cloud.google.com/tpu/docs/system-architecture



https://cloud.google.com/tpu/docs/bfloat16

#### SUPPLEMENTAL INFRINGEMENT EVIDENCE

## 7. A device comprising:

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wherein the dynamic range of the possible valid inputs to the first operation is at least as wide as from 1/1,000,000 through 1,000,000 and for at least X=5% of the possible valid inputs to the first operation, the statistical mean, over repeated execution of the first operation on each specific input from the at least X% of the possible valid inputs to the first operation, of the numerical values represented by the first output signal of the LPHDR unit executing the first operation on that input differs by at least Y=0.05% from the result of an exact mathematical calculation of the first operation on the numerical values of that same input;

at least one first computing device adapted to control the operation of the at least one first LPHDR execution unit

wherein the at least one first computing device comprises at least one of a central processing unit (CPU), a graphics processing unit (GPU), a field programmable gate array (FPGA), a microcode-based processor, a hardware sequencer, and a state machine; and,

wherein the number of LPHDR execution units in the device exceeds by at least one hundred the non-negative integer number of execution units in the device adapted to execute at least the operation of multiplication on floating point numbers that are at least 32 bits wide.

#### Systolic array

The MXU implements matrix multiplications in hardware using a so-called "systolic array" architecture in which data elements flow through an array of hardware computation units. (In medicine, "systolic" refers to heart contractions and blood flow, here to the flow of data.)

The basic element of a matrix multiplication is a dot product between a line from one matrix and a column from the other matrix (see illustration at the top of this section). For a matrix multiplication Y=X\*W, one element of the result would be:

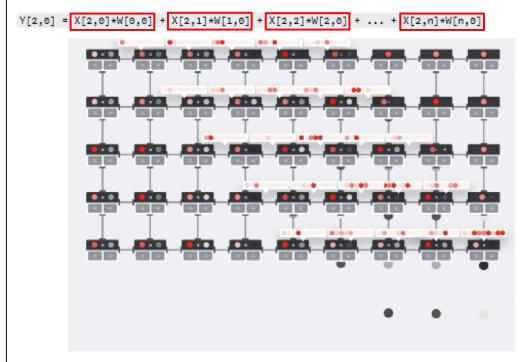


Illustration: the MXU systolic array. The compute elements are multiply-accumulators. The values of one matrix are loaded into the array (red dots). Values of the other matrix flow through the array (grey dots). Vertical lines propagate the values up. Horizontal lines propagate partial sums. It is left as an exercise to the user to verify that as the data flows through the array, you get the result of the matrix multiplication coming out of the right side.

https://codelabs.developers.google.com/codelabs/keras-flowers-convnets/#2

See also GOOG-SING-SC-000001-10, 13-30, 33-61, 228-292, 315-373, 396-444, 449-454.

#### SUPPLEMENTAL INFRINGEMENT EVIDENCE

## 7. A device comprising:

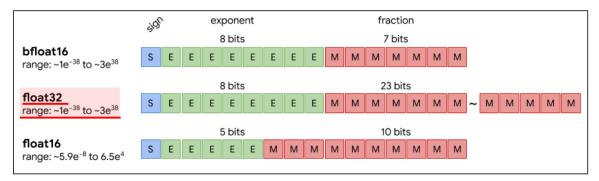
- at least one first low precision high-dynamic range (LPHDR) execution unit adapted to execute a first operation on a first input signal representing a first numerical value to produce a first output signal representing a second numerical value,
- wherein the dynamic range of the possible valid inputs to the first operation is at least as wide as from 1/1,000,000 through 1,000,000 and for at least X=5% of the possible valid inputs to the first operation, the statistical mean, over repeated execution of the first operation on each specific input from the at least X% of the possible valid inputs to the first operation, of the numerical values represented by the first output signal of the LPHDR unit executing the first operation on that input differs by at least Y=0.05% from the result of an exact mathematical calculation of the first operation on the numerical values of that same input;
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- wherein the number of LPHDR execution units in the device exceeds by at least one hundred the non-negative integer number of execution units in the device adapted to execute at least the operation of multiplication on floating point numbers that are at least 32 bits wide.

• "Each TPU core has scalar, vector, and matrix units (MXU). The MXU provides the bulk of the compute power in a TPU chip. Each MXU is capable of performing 16K multiply-accumulate operations in each cycle. While **the MXU inputs and outputs are 32-bit floating point values**, the MXU performs multiplies at reduced bfloat16 precision. Bfloat16 is a 16-bit floating point representation that provides better training and model accuracy than the IEEE half-precision representation."

https://cloud.google.com/tpu/docs/system-architecture

- "The following figure shows three floating-point[] formats
  - fp32 IEEE single-precision floating-point
  - fp16 IEEE half-precision floating point
  - bfloat16 16-bit brain floating point"

https://cloud.google.com/tpu/docs/bfloat16



Id.

See also GOOG-SING-SC-45-61, 435-444, 449-454.

#### SUPPLEMENTAL INFRINGEMENT EVIDENCE

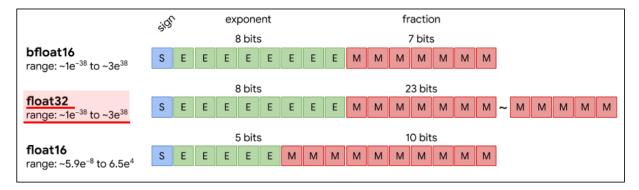
- 7. A device comprising:
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https://cloud.google.com/tpu/docs/system-architecture

- "The following figure shows three floating-point[] formats
  - fp32 IEEE single-precision floating-point
  - fp16 IEEE half-precision floating point
  - bfloat16 16-bit brain floating point"

https://cloud.google.com/tpu/docs/bfloat16



Id.

• "Because general-purpose processors such as CPUs and GPUs must provide good performance across a wide range of applications, they have evolved myriad sophisticated, performance-oriented mechanisms. As a side effect, the behavior of those processors can be difficult to predict, which makes it hard to guarantee a certain latency limit on neural network inference. In contrast, TPU design is strictly minimal and deterministic as it has to run only one task at a time: neural network prediction. You can see its simplicity in the floor plan of the TPU die."

https://cloud.google.com/blog/products/gcp/an-in-depth-look-at-googles-first-tensor-processing-unit-tpu (emphasis in orig.)

• "In mathematics, computer science and physics, a <u>deterministic</u> system is a system in which <u>no randomness</u> is involved in the development of future states of the system. A <u>deterministic</u> model will thus <u>always produce the same output</u> from a given starting condition or initial state."

https://en.wikipedia.org/wiki/Deterministic system

• For each of the possible valid inputs to the multiplication operation performed by the multipliers within the MXU, Singular has computed the result and compared it to the result of an exact mathematical calculation performed on the same inputs. The results of this test showed that for more than 10% of the possible valid inputs, the numerical value represented by the output signal of each MXU multiplier differs by more than 0.2% from the result of an exact mathematical calculation performed on the same inputs.

See also GOOG-SING-SC-000001-10, 13-30, 33-61, 228-292, 315-373, 396-444, 449-454.

#### SUPPLEMENTAL INFRINGEMENT EVIDENCE

- 7. A device comprising:
- at least one first low precision high-dynamic range (LPHDR) execution unit adapted to execute a first operation on a first input signal representing a first numerical value to produce a first output signal representing a second numerical value,
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- at least one first computing device adapted to control the operation of the at least one first LPHDR execution unit
- wherein the at least one first computing device comprises at least one of a central processing unit (CPU), a graphics processing unit (GPU), a field programmable gate array (FPGA), a microcode-based processor, a hardware sequencer, and a state machine; and,
- wherein the number of LPHDR execution units in the device exceeds by at least one hundred the non-negative integer number of execution units in the device adapted to execute at least the operation of multiplication on floating point numbers that are at least 32 bits wide.

- "Each of the cores on a TPU device can <u>execute</u> user computations (<u>XLA ops</u>) independently."

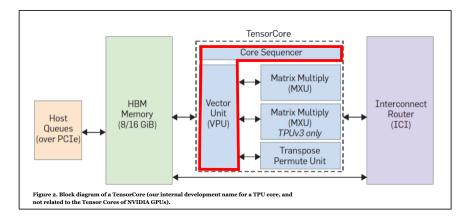
  https://cloud.google.com/tpu/docs/system-architecture#pod
- "TPUs use a VLIW architecture to express instruction-level parallelism to the many compute units of a TensorCore. XLA uses standard VLIW compilation techniques including loop unrolling, instruction scheduling, and software pipelining to keep all compute units busy and to simultaneously move data through the memory hierarchy to feed them."

https://cacm.acm.org/magazines/2020/7/245702-a-domain-specific-supercomputer-for-training-deep-neural-networks/fulltext

- "The Core Sequencer fetches VLIW (Very Long Instruction Word) instructions from the core's on-chip, software-managed Instruction Memory (Imem), executes scalar operations using a 4K 32-bit scalar data memory (Smem) and 32 32-bit scalar registers (Sregs), and forwards vector instructions to the VPU. The 322-bit VLIW instruction can launch eight operations: two scalar, two vector ALU, vector load and store, and a pair of slots that queue data to and from the matrix multiply and transpose units. The XLA compiler schedules loading Imem via independent overlays of code, as unlike conventional CPUs, there is no instruction cache."

  Id.
- "The Vector Processing Unit (VPU) performs vector operations using a large on-chip vector memory (Vmem) with 32K 128 x 32-bit elements (16MiB), and 32 2D vector registers (Vregs) that each contain 128 x 8 32-bit elements (4 KiB). The VPU streams data to and from the MXU through decoupling FIFOs. The VPU collects and distributes data to Vmem via data-level parallelism (2D matrix and vector functional units) and instruction-level parallelism (8 operations per instruction)."

  Id.



Id.

See also GOOG-SING-SC-62-227, 258-267, 269-289, 346-354, 356-373, 445-448.

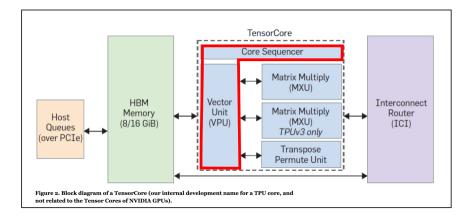
#### SUPPLEMENTAL INFRINGEMENT EVIDENCE

- 7. A device comprising:
- at least one first low precision high-dynamic range (LPHDR) execution unit adapted to execute a first operation on a first input signal representing a first numerical value to produce a first output signal representing a second numerical value.
- wherein the dynamic range of the possible valid inputs to the first operation is at least as wide as from 1/1,000,000 through 1,000,000 and for at least X=5% of the possible valid inputs to the first operation, the statistical mean, over repeated execution of the first operation on each specific input from the at least X% of the possible valid inputs to the first operation, of the numerical values represented by the first output signal of the LPHDR unit executing the first operation on that input differs by at least Y=0.05% from the result of an exact mathematical calculation of the first operation on the numerical values of that same input;
- at least one first computing device adapted to control the operation of the at least one first LPHDR execution unit;
- wherein the at least one first computing device comprises at least one of a central processing unit (CPU), a graphics processing unit (GPU), a field programmable gate array (FPGA), a microcode-based processor, a hardware sequencer, and a state machine; and,
- wherein the number of LPHDR execution units in the device exceeds by at least one hundred the non-negative integer number of execution units in the device adapted to execute at least the operation of multiplication on floating point numbers that are at least 32 bits wide.

• "The Core Sequencer fetches VLIW (Very Long Instruction Word) instructions from the core's on-chip, software-managed Instruction Memory (Imem), executes scalar operations using a 4K 32-bit scalar data memory (Smem) and 32 32-bit scalar registers (Sregs), and forwards vector instructions to the VPU. The 322-bit VLIW instruction can launch eight operations: two scalar, two vector ALU, vector load and store, and a pair of slots that queue data to and from the matrix multiply and transpose units. The XLA compiler schedules loading Imem via independent overlays of code, as unlike conventional CPUs, there is no instruction cache."

Id.

• "The Vector Processing Unit (VPU) performs vector operations using a large on-chip vector memory (Vmem) with 32K 128 x 32-bit elements (16MiB), and 32 2D vector registers (Vregs) that each contain 128 x 8 32-bit elements (4 KiB). The VPU streams data to and from the MXU through decoupling FIFOs. The VPU collects and distributes data to Vmem via data-level parallelism (2D matrix and vector functional units) and instruction-level parallelism (8 operations per instruction)."



Id.

- "Each of the cores on a TPU device can <u>execute</u> user computations (<u>XLA ops</u>) independently." https://cloud.google.com/tpu/docs/system-architecture#pod
- "TPUs use a VLIW architecture to express instruction-level parallelism to the many compute units of a TensorCore. XLA uses standard VLIW compilation techniques including loop unrolling, instruction scheduling, and software pipelining to keep all compute units busy and to simultaneously move data through the memory hierarchy to feed them."

https://cacm.acm.org/magazines/2020/7/245702-a-domain-specific-supercomputer-for-training-deep-neural-networks/fulltext

See also GOOG-SING-SC-62-227, 258-267, 269-289, 346-354, 356-373, 445-448.

#### SUPPLEMENTAL INFRINGEMENT EVIDENCE

### 7. A device comprising:

- at least one first low precision high-dynamic range (LPHDR) execution unit adapted to execute a first operation on a first input signal representing a first numerical value to produce a first output signal representing a second numerical value.
- wherein the dynamic range of the possible valid inputs to the first operation is at least as wide as from 1/1,000,000 through 1,000,000 and for at least X=5% of the possible valid inputs to the first operation, the statistical mean, over repeated execution of the first operation on each specific input from the at least X% of the possible valid inputs to the first operation, of the numerical values represented by the first output signal of the LPHDR unit executing the first operation on that input differs by at least Y=0.05% from the result of an exact mathematical calculation of the first operation on the numerical values of that same input;
- at least one first computing device adapted to control the operation of the at least one first LPHDR execution unit;
- wherein the at least one first computing device comprises at least one of a central processing unit (CPU), a graphics processing unit (GPU), a field programmable gate array (FPGA), a microcode-based processor, a hardware sequencer, and a state machine; and,
- wherein the number of LPHDR execution units in the device exceeds by at least one hundred the non-negative integer number of execution units in the device adapted to execute at least the operation of multiplication on floating point numbers that are at least 32 bits wide.

The Accused Products independently meet this claim limitation for each "device" identified above:

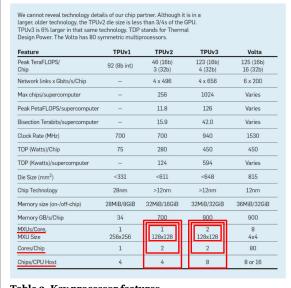
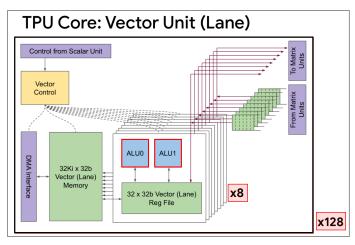


Table 3. Key processor features.

https://cacm.acm.org/magazines/2020/7/245702-a-domain-specific-supercomputer-for-training-deep-neural-networks/fulltext



Norrie et al., "Google's Training Chips Revealed: TPUv2 and TPUv3" (Presented at HotChips Conference, Aug. 2020)

See also GOOG-SING-SC-000001-454.